Composition

Composition of a function and its inverse:

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$

The composition of a function and its inverse is the **identity function** $i(x) = x$.

Graphs

The **graph** of a function $f:A \rightarrow B$ is the set of ordered pairs $\{(a, b) \mid a \in A \text{ and } f(a) = b\}$.

The graph is a subset of $A \times B$ that can be used to visualize f in a two-dimensional coordinate system.

Floor and Ceiling Functions

The **floor** and **ceiling** functions map the real numbers onto the integers (**R**$\rightarrow$**Z**).

The **floor** function assigns to $r \in$ **R** the largest $z \in$ **Z** with $z \leq r$, denoted by $\lfloor r \rfloor$.

Examples: $\lfloor 2.3 \rfloor = 2$, $\lfloor 2 \rfloor = 2$, $\lfloor 0.5 \rfloor = 0$, $\lfloor -3.5 \rfloor = -4$

The **ceiling** function assigns to $r \in$ **R** the smallest $z \in$ **Z** with $z \geq r$, denoted by $\lceil r \rceil$.

Examples: $\lceil 2.3 \rceil = 3$, $\lceil 2 \rceil = 2$, $\lceil 0.5 \rceil = 1$, $\lceil -3.5 \rceil = -3$

Now, something about

BooleanAl gebra

(section 10.1)

Boolean Algebra

Boolean algebra provides the operations and the rules for working with the set **{0, 1}**.

These are the rules that underlie **electronic circuits**, and the methods we will discuss are fundamental to **VLSI design**.

We are going to focus on three operations:

- Boolean complementation,
- Boolean sum, and
- Boolean product

Boolean Operations

The **complement** is denoted by a bar (on the slides, we will use a minus sign). It is defined by

$-0 = 1$ and $-1 = 0$.

The **Boolean sum**, denoted by + or by OR, has the following values:

$1 + 1 = 1$, $1 + 0 = 1$, $0 + 1 = 1$, $0 + 0 = 0$

The **Boolean product**, denoted by $\cdot$ or by AND, has the following values:

$1 \cdot 1 = 1$, $1 \cdot 0 = 0$, $0 \cdot 1 = 0$, $0 \cdot 0 = 0$

Boolean Functions and Expressions

Definition: Let $B = \{0, 1\}$. The variable x is called a **Boolean variable** if it assumes values only from B.

A function from B^n, the set $\{(x_1, x_2, \ldots, x_n) \mid x_i \in B, 1 \leq i \leq n\}$, to B is called a **Boolean function of degree n**.

Boolean functions can be represented using expressions made up from the variables and Boolean operations.

Boolean Functions and Expressions

The **Boolean expressions** in the variables $x_1, x_2, \ldots, x_n$ are defined recursively as follows:

- $0, 1, x_1, x_2, \ldots, x_n$ are Boolean expressions.
- If E_1 and E_2 are Boolean expressions, then $(-E_1)$, (E_1E_2), and $(E_1 + E_2)$ are Boolean expressions.

Each Boolean expression represents a Boolean function. The values of this function are obtained by substituting 0 and 1 for the variables in the expression.

Boolean Functions and Expressions

For example, we can create Boolean expression in the variables x, y, and z using the "building blocks" 0, 1, x, y, and z, and the construction rules:

Since x and y are Boolean expressions, so is xy.

Since z is a Boolean expression, so is (-z).

Since xy and (-z) are expressions, so is xy + (-z).

… and so on…

Boolean Functions and Expressions

Example: Give a Boolean expression for the Boolean function F(x, y) as defined by the following table:

x	y	F(x, y)
0	0	0
0	1	1
1	0	0
1	1	0

Possible solution: $F(x, y) = (-x)\cdot y$

Boolean Functions and Expressions

Another Example:

x	y	z	F(x, y, z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Possible solution I:

$F(x, y, z) = -(xz + y)$

Possible solution II:

$F(x, y, z) = (-(xz))(-y)$

Boolean Functions and Expressions

There is a simple method for deriving a Boolean expression for a function that is defined by a table. This method is based on **minterms**.

Definition: A **literal** is a Boolean variable or its complement. A **minterm** of the Boolean variables x_1, x_2, …, x_n is a Boolean product $y_1 y_2 \ldots y_n$, where $y_i = x_i$ or $y_i = -x_i$.

Hence, a minterm is a product of n literals, with one literal for each variable.

Boolean Functions and Expressions

Consider F(x,y,z) again:

x	y	z	F(x, y, z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

F(x, y, z) = 1 if and only if:

x = y = z = 0 or

x = y = 0, z = 1 or

x = 1, y = z = 0

Therefore,

F(x, y, z) =
(-x)(-y)(-z) +
(-x)(-y)z +
x(-y)(-z)